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Nonlinear Guided Waves: Preface

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This special issue presents a collection of experimental and theoretical research work in nonlinear waves, particularly nonlinear optics, which were presented at the conference *Nonlinear Guided Waves VIII* held in Oaxaca, Mexico in April 2016. This preface provides a short history of the conference series *Nonlinear Guided Waves* and a brief introduction to the contributed papers which puts them in context.

Keywords: nonlinear optics, solitons, solitary waves, discrete systems

1. Introduction

The conference series *Nonlinear Guided Waves* was conceived as a small, informal gathering to bring together experimental and theoretical researchers from engineering, physics, materials science and applied mathematics who work in the general area of nonlinear waves, with an emphasis on nonlinear optics. It was first held in May, 2008 at the University of Wollongong, New South Wales, Australia. Following its first edition, the conference has continuously moved to new venues. Subsequent editions have been at

(2) SASTRA University, Tamal Nadu, India, December 2008.
(3) National Autonomous University of Mexico, Mexico City, Mexico, April 2009.
(4) Cadi Ayyad University, Marrakech, Morocco, March 2010.
(5) Feza Gürsey Institute for Fundamental Sciences, Istanbul, Turkey, March 2011.
(6) University of Santiago, Santiago de Compostela, Spain, May 2012.
(7) Kingussie, Scotland, United Kingdom, May 2014.
(8) Oaxaca, Mexico, April 2016.

The conference series *Nonlinear Guided Waves* has been successful in stimulating research by people working in diverse fields, but sharing a common interest in nonlinear waves. This is apparent from the papers published in this special issue, which presents a mix and combination of experimental and theoretical work.

The study of nonlinear waves has a long history going back to the study of weakly nonlinear, periodic surface water waves by Stokes[1] nearly two hundred years ago. This work seemed to show that steady nonlinear waves are similar to steady linear waves in that they are periodic waves of progression. This view was challenged by the famous observation of Russell[2] of his "wave of translation," a "large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed." This observation of a steady, non-periodic water wave was met with scepticism, with Stokes in particular not believing that it was possible. However, the existence of a steady, non-periodic wave was set on a firm footing by the work of Boussinesq[3], Lord Rayleigh[4] and Korteweg and de Vries[5]. They showed that the "wave of translation," now termed a solitary wave, was a valid solution of the water wave equations in the weakly nonlinear, long wave limit and was due to a balance between dispersion and nonlinearity. As such, it had no linear counterpart. With the theoretical foundation of solitary waves fixed, such waves became little more than an oddity in wave research. This is shown by the brief section dealing with these waves in the classic book of Lamb collecting research on fluid mechanics and water waves[6], first published in 1879, later updated in 1932.

This situation changed dramatically with the discovery of Zabusky and Kruskal[7] in 1965 that the solitary wave solutions of the Korteweg-de Vries (KdV) equation interact "cleanly," that is they retain their shape upon interaction and the only trace of the interaction is a phase change. Due to this particle-like behaviour, the solitary wave solution of the KdV equation was termed a soliton. This numerical discovery motivated renewed research on the KdV equation and in 1967 it was found that it is exactly integrable in a Hamiltonian sense through the method of inverse scattering[8]. The method of inverse scattering showed that the nonlinear KdV equation could be reduced to a linear eigenvalue problem, the Schrödinger equation of quantum mechanics, and a linear integral equation, the Marchenko equation. This reduction of a nonlinear equation to linear equations explained the clean interactions of KdV solitons. The discovery that the KdV equation possessed an exact solution spured renewed research on nonlinear waves. It was rapidly found that many other generic wave equations, including the nonlinear Schrödinger (NLS) and Sine-Gordon equations, possessed inverse scattering solutions[9]. The other major advance in nonlinear wave theory in the 1960s was the development of modulation theory, or the method of averaged Lagrangians, by Whitham[9,10,11]. This general method, related to the applied mathematics method of multiple scales and the classical mechanics concept of adiabatic invariants, enabled the analysis of slowly varying wavetrains and, in particular, provided an ingenious approach for analysing the stability of wavetrains. One of its first successes was the explanation of the instability of deep water gravity waves, the so-called Benjamin-Feir instability[12,13].

This renewed interest in nonlinear wave theory soon spread beyond water waves and fluid mechanics, partly driven by the fact that integrable equations, such as the NLS equation, are applicable to other areas, such as optics. In 1980 it was experimentally demonstrated that an optical soliton can propagate in a glass fibre, due to a balance between the dispersion and the (small) Kerr nonlinearity of the material[14]. This confirmed a theoretical prediction based on light propagation in an optical fibre being governed by an NLS equation[15]. The application of nonlinear wave theory in optics has exploded since then, taking in both temporal and spatial waves, see, for example, the book by Kivshar and Agrawal for an overview[16]. The research published in this special issue continues and contributes to the study of nonlinear photonics and optical materials. A specific emphasis is on the nonlinear optics of "nonlocal" materials, an example being nematic liquid crystals[17]. A "nonlocal" medium is one whose optical response extends beyond the excitation wavepacket due to the material response being based on some kind of diffusive phenomenon.

2. Special issue papers

The papers in this special issue of the *Journal of Nonlinear Optical Physics and Materials* are a cross section of both temporal and spatial nonlinear optical phenomena, both continuous and discrete. Furthermore, they represent an interplay of experimental and theoretical research, which is one of the main aims of the conference *Nonlinear Guided Waves.*

The first papers deal with the nonlinear, nonlocal response of nematic liquid crystals. The optical nonlinearity of this material is non-Kerr, which allows two dimensional solitary waves to be stable and not subject to the catastrophic collapse exhibited by two dimensional solitons in Kerr media[16,17].

Highly nonlocal optical response: benefit or drawback?: The nonlocal optical response of nematic liquid crystals results in the stable propagation of $(2+1)$ dimensional solitary waves in this bulk medium. The advantages and disadvantages of the nonlocal response of nematic liquid crystals from both experimental and theoretical viewpoints are discussed and contrasted. It is found that, as well as stabilising $(2+1)$ dimensional solitary waves, the nonlocality stabilises other optical structures such as vortices which are unstable in media with local responses.

An additional benefit of a nonlocal response is that optical beams can interact "at a distance," that is, through the light-induced medium distortion. This long-range interaction [18] can be exploited to create circuits and switches. While the positives of a nonlocal medium appear to outweigh the negatives, such a response does have drawbacks for some optical applications, mainly due to the smoothing of fine features. In addition, the overall response time of nonlocal media tends to be slow.

Beam on beam control: beyond the particle approximation: The nonlocal response of nematic liquid crystals and its implications for the interaction of two nematicons at a distance is taken up in detail in this paper. This work considers a basic logic circuit whereby a "control" beam can route a "signal" beam to a continuum of output positions at the end of the liquid crystal cell. This routing is studied via both full numerical solutions of the governing nematic equations and an asymptotic reduction obtained using "momentum conservation", assuming that the nonlinear beams can be approximated by "point particles." It is found that this approximation yields only adequate agreement with numerical solutions. The "momentum conservation" approach is then extended to account for the finite width of optical beams, resulting in excellent agreement with numerical solutions.

Nonlinear optical properties of dielectric nanocolloids: particle size and concentration effects: This paper reports an experimental study of optical solitary waves in a medium consisting of water with suspended nano-particles. These particles change the optical response so that the medium is focusing and can support bright optical solitary waves. An unexpected result is that this colloidal medium has indications of nonlocality, in spite of the current theoretical models which pinpoint a local response for colloidal media[19]. The experiments highlight further issues of interest from both experimental and theoretical points of view, including thermal convection. The effect of convective fluid motion on an optical field is unexplored.

Circular dispersive shock waves in colloidal media: This paper deals with beam propagation in a colloidal medium, but considers an optical dispersive shock wave, or optical undular bore, rather than a solitary wave. A dispersive shock wave is the dispersive equivalent of a shock wave in compressible fluid flow as it connects two distinct (optical intensity) levels. The main difference between a compressible flow shock and a dispersive shock is that the former is a discontinuous jump, while the latter is a smooth, continually expanding modulated wavetrain linking the two levels. This paper shows that while wavetrains are unstable in a focusing colloidal medium, undular bores can exist for a finite propagation distance before becoming unstable[21]. In addition, it considers radially symmetric two space dimensional dispersive shock waves, a largely unexplored area[20].

Shelf solutions and dispersive shocks in a discrete NLS equation: effects of nonlocality: This paper deals with dispersive shock waves, but in a discrete, defocusing, nonlocal, nonlinear medium governed by a discrete version of the equations for nematic liquid crystals[22]. A dispersive shock wave is generated by a jump of the initial optical intensity and studied in the strong linear inter-site coupling limit. It is found that the discrete, nonlocal dispersive shock wave has many features of the equivalent continuous, local NLS dispersive shock wave and can be approximately described by continuous NLS hydrodynamics[20]. The nonlocal response is found to play a role in the development of the dispersive shock wave, as its smaller scale oscillations are greaty smoothed compared with the equivalent discrete NLS dispersive shock wave.

Light dynamics in nonlinear trimmers and twisted multicore fibers: This paper on discrete NLS equations considers arrays of coupled $\mathcal{PT}$ symmetric optical fibres with a few nodes, specifically two and three nodes, and twisted multicore fibres. This work extends previous work by the inclusion of nonlinear effects[23].

$\mathcal{PT}$ symmetric equations have gain and loss in balance, so that stable waves can propagate. The solutions of such equations have properties not observed in their conservative equivalents. In particular, for a three waveguide array it is found that there exists a parametric transition region from unbroken to broken $\mathcal{PT}$ symmetry. A nonlinear fibre with six cores and an induced twist is also considered, a case differing from the usual $\mathcal{PT}$ symmetric case in that the gain/loss is not continuous, but alternate in balance. These results for a twisted fibre suggest the possibility of controlling light dynamics in multicore fibres.

Coherent states and localization in a quantized discrete NLS lattice: This is a numerical study of quantised discrete NLS equations, focusing on the effect of quantisation on stable and unstable breather solutions, a largely unexplored area. It is found that the quantum breather equivalent of a stable breather oscillates between a classical state and a quantum coherent state with a short oscillation distance. On the other hand, unstable breathers show no recurrence, but have a large distance between the classical and quantum states.

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